

# TRANSVERSITY SINGLE SPIN ASYMMETRIES\*

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The theoretical aspects of two leading twist transversity single spin asymmetries, one arising from the Collins effect and one from the interference fragmentation functions, are reviewed. Issues of factorization, evolution and Sudakov factors for the relevant observables are discussed. These theoretical considerations pinpoint the most realistic scenarios towards measurements of transversity.

## 1 Collins effect asymmetries

The Collins effect refers to a nonzero correlation between the transverse spin  $\mathbf{s}_T$  of a fragmenting quark and the distribution of produced hadrons. More specifically, a transversely polarized quark can fragment into particles (with nonzero transverse momentum  $\mathbf{k}_T$ ) having a  $\mathbf{k}_T \times \mathbf{s}_T$  angular distribution around the jet axis or, equivalently, the quark momentum, see Fig. 1. The

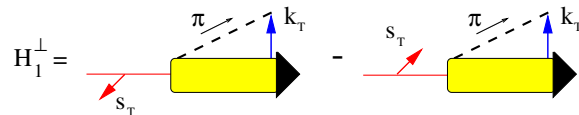


Figure 1. The Collins effect:  $q(\mathbf{s}_T) \rightarrow \pi(\mathbf{k}_T)X \neq q(-\mathbf{s}_T) \rightarrow \pi(\mathbf{k}_T)X$ .

Collins effect will be denoted by a fragmentation function  $H_1^\perp(z, \mathbf{k}_T)$  and is expected to be nonzero due to final state interactions between a measured final state hadron (e.g. a  $\pi$ ) and the rest of the jet ( $X$ ). The Collins effect can lead to single spin asymmetries (SSA) in  $e p^\uparrow \rightarrow e' \pi X$  and  $p p^\uparrow \rightarrow \pi X$ . There exist some experimental indications that the Collins effect is nonzero, e.g. SSA measured by HERMES<sup>1,2</sup> and SMC<sup>3</sup> at relatively low energies.

### 1.1 Collins effect in semi-inclusive DIS

Collins<sup>4</sup> considered semi-inclusive DIS  $e + p^\uparrow \rightarrow e' + \pi + X$ , where the spin of the proton is orthogonal to the direction of the virtual photon  $\gamma^*$  and one observes the transverse momentum  $\mathbf{P}_\perp^\pi$  of the  $\pi$  in the jet, which has an angle  $\phi_\pi^e$  compared to the lepton scattering plane ( $\phi$  in Fig. 2). Collins has shown

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Here we will briefly consider the effects of Sudakov factors in the explicit example of the Collins effect asymmetry in semi-inclusive DIS  $e p \rightarrow e' \gamma^*(q_T) p \rightarrow e' \pi X$  ( $q_T = -z P_\perp^\pi$  and  $q_T^2 \equiv Q_T^2 \ll Q^2$ )

$$\frac{d\sigma(e p \rightarrow e' \pi X)}{dx dz dy d\phi_e d^2 q_T} \propto \{1 + |\mathbf{S}_T| \sin(\phi_\pi^\epsilon + \phi_S^\epsilon) A(q_T)\}. \quad (4)$$

To get an idea about the effect of Sudakov factors, we will assume Gaussian transverse momentum dependence for  $H_1^\perp$ . The asymmetry analyzing power is then given by

$$A(q_T) = \frac{\sum_a e_a^2 B(y) h_1^a(x) H_1^{\perp(1)a}(z)}{\sum_b e_b^2 A(y) f_1^b(x) D_1^b(z)} \mathcal{A}(Q_T), \quad (5)$$

where  $A(y) = (1 - y + \frac{1}{2}y^2)$ ,  $B(y) = (1 - y)$ . Furthermore, since the nonperturbative Sudakov factor ( $S_{NP}$ ) is not determined from SIDIS experiments (despite the ZEUS data), for illustration purposes we will use the parameterization of Ladinsky-Yuan<sup>10</sup>. In Fig. 3  $\mathcal{A}(Q_T)$  is given at  $Q = M_Z$  and compared to the tree level result for Gaussian transverse momentum widths chosen such as to minimize that result (values more typical of a tree level analysis produce a larger asymmetry factor). We refer to Ref.<sup>9</sup> for details. We see that  $\mathcal{A}(Q_T)$

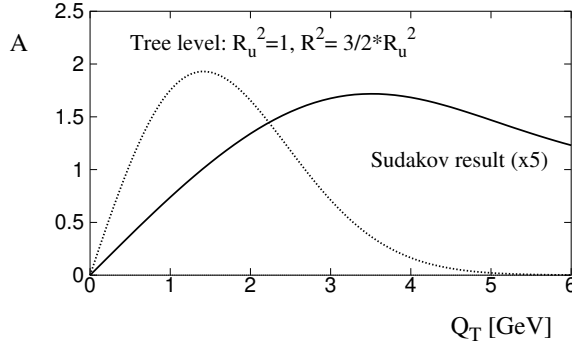


Figure 3. The asymmetry factor  $\mathcal{A}(Q_T)$  (in units of  $M_h$ ). The solid curve is a generic Sudakov factor result (at  $Q = 90$  GeV) multiplied by a factor 5. The other curve is the tree level quantity for certain Gaussian transverse momentum widths.

at  $Q = M_Z$  becomes considerably smaller ( $\max(\mathcal{A}(Q_T)) \sim Q^{-0.5} - Q^{-0.6}$ ) and broader than the tree level expectation. Thus, tree level estimates tend to overestimate transverse momentum dependent azimuthal spin asymmetries and Sudakov factors cannot be ignored at present-day collider energies.

### 1.2 Collins effect in $e^+ e^- \rightarrow \pi^+ \pi^- X$

In order to obtain the Collins function, one can measure a  $\cos(2\phi)$  asymmetry in  $e^+ e^- \rightarrow \pi^+ \pi^- X$ , that is essentially proportional to the Collins function squared<sup>11</sup> (at average momentum fractions). A first indication of such a

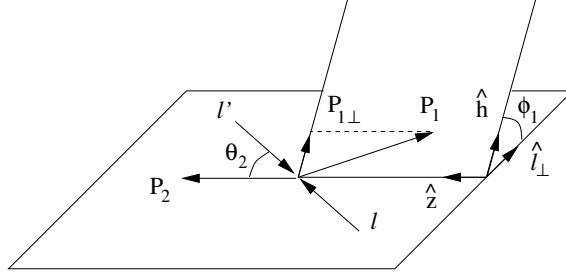


Figure 4. Kinematics of  $ll' \rightarrow P_1 P_2 X$ , where  $l, l'$  are the momenta of  $e^-, e^+$ ;  $P_1, P_2$  are the momenta of hadrons in opposite jets.

nonzero asymmetry comes from a preliminary analysis<sup>12</sup> of the 91-95 LEP1 data (DELPHI). Also, the possibilities at BELLE are currently being examined<sup>13</sup>. The extraction of the Collins function from this asymmetry is not straightforward, since there is asymmetric background from hard gluon radiation (with  $Q_T \sim Q$ ) and from weak decays. Moreover, using the tree level asymmetry expression is not sufficient and beyond tree level Sudakov factors need to be included. If the differential cross section is written as

$$\frac{d\sigma(e^+ e^- \rightarrow \pi^+ \pi^- X)}{d\Omega dz_1 dz_2 d^2 \mathbf{q}_T} \propto \{1 + \cos(2\phi_1) A(\mathbf{q}_T)\}, \quad (6)$$

with  $\mathbf{q}_T^2 \ll Q^2$ , then assuming again Gaussian transverse momentum dependence for the Collins function, we find

$$A(\mathbf{q}_T) = \frac{\sum_a c_2^a B(y) H_1^{\perp(1)a}(z_1) \overline{H}_1^{\perp(1)a}(z_2)}{\sum_a c_1^a A(y) D_1^a(z_1) \overline{D}_1^a(z_2)} \mathcal{A}(Q_T), \quad (7)$$

with somewhat different  $A(y), B(y)$ . Surprisingly there is also no determination of  $S_{NP}(b)$  from  $e^+ e^- \rightarrow A B X$ , so for illustration purposes we use again Ladinsky-Yuan's  $S_{NP}(b)$ . The result is displayed in Fig. 5 and compared to a conservative (i.e. expected to be too small) tree level curve. This generic example<sup>9</sup> shows that Sudakov factors produce an order of magnitude suppression at  $Q = M_Z$  ( $\max(\mathcal{A}(Q_T)) \sim Q^{-0.9} - Q^{-1.0}$ ), hence this Collins effect observable is best studied with two jets events at  $\sqrt{s} \ll M_Z$ .

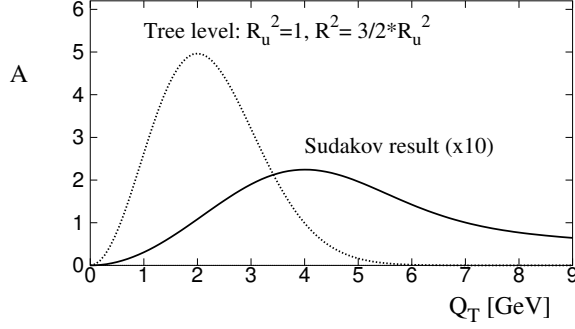


Figure 5. The asymmetry factor  $\mathcal{A}(Q_T)$  (in units of  $M^2$ ). The solid curve is a generic Sudakov factor result (at  $Q = 90$  GeV) multiplied by a factor 10. The other curve is the tree level quantity for certain Gaussian transverse momentum widths.

## 2 Interference fragmentation functions

Jaffe, Jin and Tang<sup>14</sup> pointed out the possibility that the Collins effect averages to zero in the sum over final states  $X$ . Instead, they proposed to measure two pions in the final state  $|(\pi^+ \pi^-)_{\text{out}} X\rangle$  ( $\pi^+, \pi^-$  belong to the same jet), which presumably depends on the strong phase shifts of the  $(\pi^+ \pi^-)$  system. The interference between different partial waves could give rise to a nonzero chiral-odd fragmentation function called the interference fragmentation function (IFF). The IFF would lead to single spin asymmetries in  $e p^\uparrow \rightarrow e' (\pi^+ \pi^-) X$  and  $p p^\uparrow \rightarrow (\pi^+ \pi^-) X$ , both proportional to the transversity function. The SSA expression for  $e p^\uparrow \rightarrow e' (\pi^+ \pi^-) X$  is<sup>14</sup>

$$\langle \cos(\phi_{S_T}^e + \phi_{R_T}^e) \rangle \propto |\mathbf{S}_T| |\mathbf{R}_T| F(m^2) \sum_{a, \bar{a}} e_a^2 x h_1^a(x) \delta \hat{q}_I(z), \quad (8)$$

where  $z = z^+ + z^-$ ;  $\mathbf{R}_T = (z^+ \mathbf{k}^- - z^- \mathbf{k}^+)/z$ ;  $m^2$  is the  $\pi^+ \pi^-$  invariant mass; and  $F(m^2) = \sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1)$ , where  $\delta_0, \delta_1$  are the  $\ell = 0, 1$  phase shifts. Note the implicit assumption of factorization of  $z$  and  $m^2$  dependence, which leads to the prediction that on the  $\rho$  resonance the asymmetry is zero (according to the experimentally determined phase shifts). But more general  $z, m^2$  dependences have been considered<sup>15</sup> and should be tested.

The asymmetry expression is based on a collinear factorization theorem (soft gluon contributions cancel, no Sudakov factors appear). Theoretically this is very clean and an analysis beyond tree level is conceptually straightforward. The evolution of  $\delta \hat{q}_I(z)$  is the same as for the transversity *fragmentation*

function  $H_1(z)$ , e.g. the LO evolution equation is given by

$$\frac{\partial z\delta\hat{q}_I(z)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} C_F \int_z^1 dy \left[ \frac{3}{2} \delta(y-z) + \frac{2z}{y(y-z)_+} \right] y\delta\hat{q}_I(y). \quad (9)$$

A NLO analysis is feasible and analogous to  $A_{TT}^{DY} \propto \cos(\phi_{S_{1T}}^e + \phi_{S_{2T}}^e) h_1 \bar{h}_1$ .

For the extraction of the interference fragmentation functions one can study a  $\cos(\phi_{R_{1T}}^e + \phi_{R_{2T}}^e)$  asymmetry<sup>16</sup> in  $e^+ e^- \rightarrow (\pi^+ \pi^-)_{\text{jet 1}} (\pi^+ \pi^-)_{\text{jet 2}} X$  which is proportional to  $(\delta\hat{q}_I)^2$ . This is again possible at BELLE and in this case there is no expected asymmetric background. Combining such a result with for instance the single spin asymmetry in  $pp^\uparrow \rightarrow \pi^+ \pi^- X$  to be measured at RHIC, seems –at present– to be one of the most realistic ways of obtaining information on the transversity function.

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